Combining Logics for Multi-agent Systems

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February 10, 1997

This document contains some preliminary ideas about the application of Combining Logics to Agents Theory.

1 Introduction

In the last twenty years or so, many logic systems have been proposed to model particular aspects of agenthood, e.g. beliefs, knowledge, actions, intentions, goals, etc.. Nevertheless, the design of agents and multi-agents systems (MAS) in general require all these mental phenomena to exist in a single system.

Rather than building new systems for dealing with multiple aspects of agenthood, the idea of *combining different logics* seems a strategy worth investigating for modelling multi-agents systems.

2 Requirements of a formalism for MAS

Agents are complex objects: they have goals, intentions, beliefs, obligations, and they perform actions (co-operatively or otherwise) in a society of other agents. Several logics have been proposed for describing MAS; for example,

- logics of knowledge and belief ([HM92], etc.) for describing agents' information states;
- logics for goals and intentions ([CL90, Woo94], etc.) for describing agents' pro-attitudes;
- formalisms for representing communication acts ([CM86], etc.).
- temporal logic [vB83], for specifying how an agent will change over time;

These formalisms have been successful in dealing with the particular aspect of agents that they address. But generally, they ignore the other aspects, and sometimes just consider the situation from the point of view of one agent. A formalism for a multi-agent system must reflect all those aspects of agents, and the agent multiplicity. A few logics have emerged which attempt to deal with several aspects at the same time, and/or address several agents [vLvdHM96, FHMV95, KL88, RG91, Kro96, Wag96, Woo96]. However, we are still some way off having a logic which addresses all the features.

3 Combining logics and MAS

Building a logic addressing all the aspects of MAS is a monolithic and error-prone task. Rather than build such a logic from scratch, we believe that it is better to study how the existing logics dealing with individual aspects may be combined. Indeed, the field of *combining logics* is emerging as an active area, promising powerful results such as the preservation of important properties of the logics being combined [Gab92, Gab, KW91, dR92, BdR].

The problem of combining logics is this: given two logics $\mathcal{A}$ and $\mathcal{B}$, how to combine them into a single logic $\mathcal{A} \otimes \mathcal{B}$ which extends the expressive power of each one? For example, suppose $\mathcal{A}$ addresses temporal aspects of agents and $\mathcal{B}$ addresses epistemic aspects. Their combination should be able to express both temporal and epistemic properties, but also the interaction of these two aspects: evolving knowledge, and knowledge about a changing world.

Several techniques for combining logics have been proposed, having different properties. Let L_1 and L_2 be two logics, having the normal modalities $\Box_1$ and $\Box_2$ respectively (as well as the usual boolean connectives). The *language* of the combination is usually the entire set of formulas which can be constructed from the boolean connectives and $\Box_1$ and $\Box_2$. This includes the languages of L_1 and L_2, and also (for example) $\Box_1\Box_2 p \to \Box_2 p$ and $\Box_1 p \to \Box_2 p$. The *semantics* of the combination may be roughly characterised as follows:

Fusion. [KW91] consider the fusion of L_1 and L_2, being 'the least bimodal logic containing both'. They show that several properties of the component logics transfer to the combination, such as completeness, fmp, interpolation. However, the fusion of two logics doesn't satisfy any interaction between the two modalities, such as the interaction expressed by the formula $\Box_1 p \to \Box_2 p$.

Embedding. Finger/Gabbay [FG92] describe a technique in which a logic is embedded inside a temporal logic. According to this method, the language is restricted: $\Box_2$ of the temporal logic may not occur in the scope of a $\Box_1$ from the other logic. A model of the combination consists of a model of the temporal logic, together with a map from each time point to a model of the other logic. More generally, any two logics can be combined by embedding one inside the other. This is an asymmetric combination: the "outer logic" can talk about formulas of the "inner" one, but not conversely.

Full-fibring. A more symmetric combination is provided by Gabbay's 'full-fibring' technique [Gab, Gab92, BdR]. In that work, formulas of the combined logic can mix operators from the component logics arbitrarily, and the semantics is defined by going back and forth between models of the component logics.

Full-join. In this approach [FG] (called the '$T \times W$ approach' in [Tho84]) Kripke frames of the combined system are of the form $(W_1 \times W_2, R_1 \times =, R_2 \times =)$, where (W_i, R_i) is an L_i frame. This approach satisfies many formulas expressing interaction between the two modalities, such as $\Box_1\Box_2 p \leftrightarrow \Box_2\Box_1 p$.

Join. The 'fusion' and 'full-join' approaches represent the two ends of a spectrum, along which different degrees of interaction are satisfied. Intermediate positions along the spectrum may be specified by taking as frames (W, R_1, R_2), and imposing certain extra conditions. Fusion occurs when there are no extra conditions; full-join occurs when the conditions are (among others): if xR_1y and xR_2z then exists w s.t. yR_2w and zR_1w.

There are more techniques in the literature.

We believe these techniques can be applied to building logics to model MAS. To motivate our claim we briefly explore some known logics and ask whether they can be seen as combinations of simpler logics. We believe that such examples should be used to evaluate the techniques for combining logics, and to guide the future development of the area.

4 Examples

- **Extension to n agents.** KD45n [HM92] is a generalisation to n agents of the modal logic KD45 used for modelling beliefs.

  Question: Is KD45n a **fusion** of KD45 with itself, n times?

  This would provide a general technique for extending an arbitrary modal logic to n agents.

- **Knowledge and time.** One one hand, the culmination of decades of MAS research by Halpern, Moses and others on epistemic logic and its interaction with temporal logic has appeared as [FHMV95]. They consider systems for modelling perfect recall, synchronicity, asynchronicity, etc. On the other hand, the **embedding** and **full-fibring** techniques have been designed explicitly for temporalising modal logics, such as epistemic logic.

  Question: To what extent can systems in [FHMV95] be seen as **embedding** non-temporal epistemic logics within temporal logic?

  Such an embedding will allow us to consider evolving knowledge. The dual embedding (temporal logic within epistemic logic) provides the ability to reason about knowledge in a changing world.

  A positive answer to this question will help validate the embedding technique as a tool for temporalising any static aspect of agents.

  Question: What sort of **join** of the epistemic logic KT45 and a temporal logic are these systems?

- **Quantified logics for MAS.** Much work has taken place to tailor predicate modal logics for some intensional aspects of agents (eg, beliefs [LC96], knowledge [LM94]). But, as [BdR] point out, this should be seen as a combination of modal logic and predicate logic. Since the language should allow arbitrary nesting of the quantifiers and the modalities, we should ask:

  Question: Are standard systems of quantified modal logic **full-fibring** combinations of modal and predicate logics?

  This would give us a systematic way of developing highly expressive systems for specifying MAS.

5 Outlook

The techniques for combining logics are theoretically general and powerful, but their applicability needs to be explored. MAS provides an excellent test-bed which should be used to guide the development of further techniques. Combining logics could provide a systematic attack on the problem of relating different aspects of agency in a formalism for MAS.

Answers to the questions in the preceding section will help evaluate the claim that the emerging field of combining logics can yield rigorous and expressive formalisms for MAS, in a general and systematic way.

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