Anonymity Protocol with Identity Escrow and Analysis in the Applied pi-calculus

Aybek Mukhamedov and Mark D. Ryan

School of Computer Science
University of Birmingham
{A.Mukhamedov,M.D.Ryan}@cs.bham.ac.uk

Abstract. *Anonymity with identity escrow* attempts to allow users of a service to remain anonymous, while providing the possibility that the service owner can break the anonymity in exceptional circumstances, such as to assist in a criminal investigation. A protocol for achieving anonymity with identity escrow has been presented by Marshall and Molina-Jiminez. In this paper, we show that protocol suffers from some serious flaws and present an improved protocol which fixes these flaws. Our improved protocol guarantees anonymity even if all but one of the escrow holders are corrupt. We analyse the protocol in the applied pi-calculus.

1 Introduction

With the increasing sophistication and adoption of communication systems in businesses and personal use, privacy and anonymity has become a concern among users [2, 11, 1]. Service usages (such as usage of mobile phones, internet, financial payments) are routinely logged, and those logs will allow organisations to build sophisticated profiles of customers and their preferences and associates. Users fear that this information could be abused. But while users may wish for complete privacy and anonymity, the failure of digital cash to achieve widespread adoption shows that society as a whole also requires security and accountability. Digital cash failed because it would allow criminal behaviour to go undetected. An appropriate balance between unrestricted anonymity and totalitarian security needs to be found, and this is likely to be a major theme in security research for some years.

Identity escrow attempts to provide such a balance for some applications. It allows users to use services anonymously while guaranteeing that service providers can break the anonymity in special circumstances; for example, to assist in a criminal investigation. If Alice wishes to use the service from provider S, she first puts her identity in escrow with a *escrow agent* T, from whom she obtains a token. She presents the token to S as evidence that she has placed her identity in escrow. S allows her to use the service anonymously. In the event of service misuse, S can apply to T to obtain the identity of the user corresponding to the token.

Identity escrow systems were first introduced by Kilian and Petrank in [14], which was motivated by the ideas from *key escrow encryption systems* (e.g. [16, 19]). *Group signature schemes* and *anonymous credential systems* are two mechanisms which can be used to offer identity escrow [8–10, 13]. In both cases, a single agent (known as *group manager* or *issuer*) holds the escrowed identity. Clearly, the system breaks down if the escrow agent is dishonest, and reveals Alice's identity even if the agreed conditions for doing so have not been met. To address this problem we propose a protocol in which the escrow agent is implemented as a *set* of agents called *token providers*. Neither S nor any token provider are supposed to know the identity behind an escrowed certificate, but if it is proved necessary, all token providers can cooperate in order to reveal it. This aims to provide a much stronger security property than group signatures or credential systems. Alice can choose the set of token providers she uses, and the idea is that her anonymity is preserved provided at least one of them is honest. Another advantage of our scheme is that it is based on standard cryptographic primitives (digital signatures and public-key encryption), which are better known than zero-knowledge proofs and more widely implemented in APIs.

Our scheme offers several advantages over identity escrow schemes mentioned above. The esrowed identity is distributed among several token providers chosen by the user. Moreover, the user's list (except for the last token provider in the list) is not revealed when the user presents her token to the service provider. Lastly, our scheme uses standard cryptographic primitives, which can be chosen in a way as not to rely on esoteric hardness assumptions such as the strong RSA assumption or the Lysyanskaya, Rivest, Sahai, and Wolf (LRSW) assumption [17]. We model and analyze our protocol in the applied pi-calculus, and show that it satisfies the anonymity property.

In an earlier work Marshall and Molina-Jiminez [18] proposed a protocol that aimed to achieve goals similar to ours – distribute escrowed user identity among several trusted parties using standard cryptographic primitives. However, their scheme requires all the trusted parties to be trusted and suffers from serious flaws, which we show in the paper.

The paper is organised as follows. In the following two sections, we present our protocol together with its formal analysis in the applied pi-calculus. Next, we briefly detail Marshall and Molina-Jiminez's protocol and the problems we have identified with it, and conclude in section 5. Appendix A summarises the applied pi-calculus, for the benefit of readers not familiar with it. Appendix B contains the proofs of lemmas we rely upon for our analysis.

Our protocol together with its formal analysis in applied pi-calculus appeared in [21] and our analysis of Marshall and Molina-Jiminez's protocol apeared in [20].

2 The Protocol

Notation. The following labeling conventions are used throughout the paper:

- S denotes an anonymous service provider.

- $T = \{T_1, T_2, \ldots T_n\}$ is a set of *identity token providers*.
- Φ_i is an identity token issued by T_{a_i}. We write Φ_A for the full identity token obtained by A by using the protocol.
- K_A is A's long-term public key. $K_{[A]}$ is the public part of A's ephemoral key that it uses to access anonymous services provided by S. $K_{[A]}$ is freshly generated by A for each service usage and no other agent knows the correspondence between $K_{[A]}$ and A.
- $\{m\}_K$ is the message m deterministically encrypted with the public key K and $[m]_{K^-}$ is A's universally verifiable signature on m.

Our protocol consists of two parts. First, there is a *sign-up* protocol, which is the main protocol that is executed by A to receive a token from the members of T. The token permits A to use the service from S. Next, there is a *complaint resolution* protocol, which is executed by S upon a misuse of its service, in order to reveal the identity of the offending anonymous user.

Sign-up. Alice has a long-term certified public key K_A. A creates a temporary service public key $K_{[A]}$ which she will use to identify herself to S.

Alice starts by building up an onion Φ_n. At the end of this process (actions 1, 2 below), the onion consists of the service key $K_{[A]}$ in its centre, wrapped with encryptions and signatures by the token providers. This is then paired with Alice's identity A, and by engaging in the protocol with token providers, it is wrapped again by encryptions and signatures of the token providers. The formal definition follows; an illustration in the case $n = 4$ is given in Figure 1.

A choses a sequence of token providers $T_{a_1}, T_{a_2}, \ldots, T_{a_n}$ from T (possibly with duplications) and creates the following term:

$$1) \qquad \Phi_1 = \{\ \texttt{InitITKReq}, K_{[A]},\ \texttt{K}_{\texttt{a}_1}\ \}_{K_{T_{a_1}}}$$

It is an encryption of a tag `InitITKReq`, public part of A's service key $K_{[A]}$ and the symmetric key $\texttt{K}_{a_1}$ by T_{a_1}'s public key, where $\texttt{K}_{a_1}$ is freshly generated by A. The goal of the protocol is to have the key $K_{[A]}$ associated with A's identity token in a way that does not reveal this link even if all but one T_{a_i} are dishonest.

Next A creates further terms from Φ_1:

$$2) \text{ for } i = 2 \text{ to } n-1 \quad : \qquad \Phi_i = \{\ \texttt{ITKReq}, \Phi_{i-1}, \texttt{K}_{\texttt{a}_i}\ \}_{K_{T_{a_i}}}$$

By the end of the sequence of encryptions (2) ($n-2$ times), A will have obtained the token Φ_{n-1}:

$$\{\ \texttt{ITKReq}, \{\ \ldots \\ \{\ \texttt{ITKReq}, \{\texttt{InitITKReq}, K_{[A]}, \texttt{K}_{a_1}\}_{K_{T_{a_1}}}, \texttt{K}_{a_2}\}_{K_{T_{a_2}}} \\ \ldots\ \}_{K_{T_{a_{n-2}}}}, \texttt{K}_{a_{n-1}}\}_{K_{T_{a_{n-1}}}}$$

The token Φ_{n-1} serves as a disguise of the service key $K_{[A]}$. The symmetric keys $\texttt{K}_{\texttt{a}_i}$ generated by A in the above steps are used in order to randomise the ciphertexts and to encrypt messages from token providers in later stages.

Next, A signs the token Φ_{n-1} and sends it to T_{a_n}, and then contacts $T_{a_{n-1}}$, $T_{a_{n-2}}, \ldots, T_{a_1}$ anonymously as shown in Steps 3, 4, 3a, 4a. They reverse the sequence of encryptions, and at the same time build up the identity token $\widetilde{\Phi}_{n-1}$. The notation $A \longmapsto B$ means that A anonymously sends a message to B. In this case, B does not know A's identity. Similarly, $A \longrightarrow\!\!| \; B$ means that B receives a message anonymously, from A; A does not know B's identity.

$$3)\quad A \quad \longmapsto \quad T_{a_n} \quad : \quad \{\, [\, \mathtt{InitITKSig},\ \Phi_{n-1},\ A\,]_{K_A^-}\, \}_{K_{T_{a_n}}}$$

$$4)\quad T_{a_n} \quad \longrightarrow\!\!| \quad A \quad : \quad \widetilde{\Phi}_1$$
$$\text{where } \widetilde{\Phi}_1 = [\, \{\, [\, \mathtt{InitITKSig},\ \Phi_{n-1},\ A\,]_{K_A^-}\, \}_{K_{T_{a_n}}}, \Phi_{n-1}\,]_{K_{T_{a_n}}^-}$$

For $i = 1$ to $n-2$:

$$* \begin{cases} 3a)\quad A \quad \longmapsto \quad T_{a_{n-i}} \quad : \quad \{\, \mathtt{ITKSig}, \widetilde{\Phi}_i, \mathtt{K}_{\mathtt{a_{n-i}}}\, \}_{K_{T_{a_{n-i}}}} \\ \text{where for } i > 1 \quad \widetilde{\Phi}_i = [\, \{\, \widetilde{\Phi}_{i-1}, \mathtt{K}_{\mathtt{a_{n-i+1}}}\}_{K_{T_{a_{n-i+1}}}},\ \Phi_{n-i}\,]_{K_{T_{a_{n+1-i}}}^-} \\ \\ 4a)\quad T_{a_{n-i}} \quad \longrightarrow\!\!| \quad A \quad : \quad \{\, [\, \{\, \widetilde{\Phi}_i, \mathtt{K}_{\mathtt{a_{n-i}}}\, \}_{K_{T_{a_{n-i}}}}, \Phi_{n-i-1}\,]_{K_{T_{a_{n-i}}}^-}\, \}_{\mathtt{K}_{\mathtt{a_{n-i}}}} \end{cases}$$

After step 3a, before sending out a response, the token provider $T_{a_{n-i}}$ checks that the session key $\mathtt{K}_{\mathtt{a_{n-i}}}$ supplied in the request matches the one embedded in Φ_{n-i} (cf. step 2 above). The same rule applies to T_{a_1} at step 5. In addition, both token providers also check that $\widetilde{\Phi}_i$ contained in the request was signed by a token provider.

$$5)\quad A \quad \longmapsto \quad T_{a_1} \quad : \quad \{\, \mathtt{ITKSig}, \widetilde{\Phi}_{n-1}, \mathtt{K}_{\mathtt{a_1}}\}_{K_{T_{a_1}}}$$

$$6)\quad T_{a_1} \quad \longrightarrow\!\!| \quad A \quad : \quad \{\, \widetilde{\Phi}_A\, \}_{\mathtt{K}_{\mathtt{a_1}}},$$
$$\text{where } \widetilde{\Phi}_A = [\, \{\, \widetilde{\Phi}_{n-1}, \mathtt{K}_{\mathtt{a_1}}\, \}_{K_{T_{a_1}}}, K_{[A]}\,]_{K_{T_{a_1}}^-}$$

Upon reaching step 6, A has the following identity token:

$$\widetilde{\Phi}_A = [\, \{\, \ldots\ [\, \{\widetilde{\Phi}_1,\ \mathtt{K}_{\mathtt{a_{n-1}}}\}_{K_{T_{a_{n-1}}}}, \Phi_{n-2}\,]_{K_{T_{a_{n-1}}}^-} \ldots \mathtt{K}_{\mathtt{a_1}}\}_{K_{T_{a_1}}}, K_{[A]}\,]_{K_{T_{a_1}}^-}$$

The token $\widetilde{\Phi}_A$ associates the service key $K_{[A]}$ with A. He presents the token to S when requesting its service.

$$7)\quad A \quad \longmapsto \quad S \quad : \quad \{\, \widetilde{\Phi}_A,\ K_{[A]}\, \}_{K_S}$$

S checks that the token is signed by some token provider and the key $K_{[A]}$ is embedded in it. In case of service misuse the token may be delayered to reveal the identity of a user bound to it via the complaint resolution protocol.

Complaint Resolution We assume that some misuse evidence $\widetilde{\Psi}_{K_{[A]}}$ is uniquely associated with A's service key $K_{[A]}$ and the service provider S. It must be verifiable by each token provider (or endorsed by a third party accepted by all token providers) and not forgeable by S.

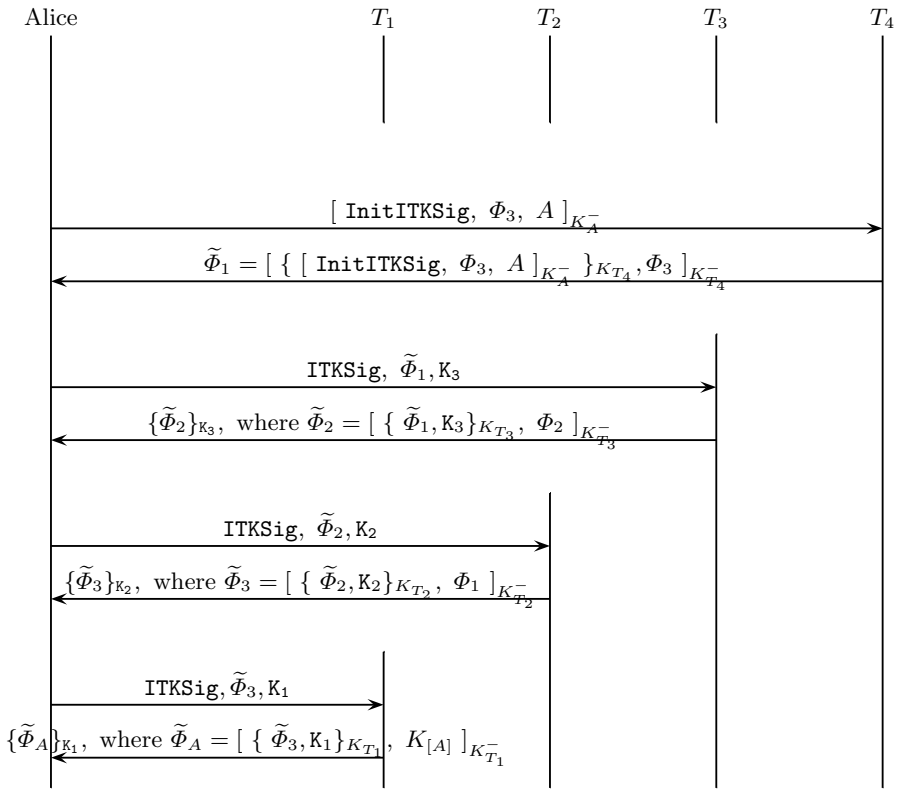

Fig. 1. Illustration of the sign-up protocol in the case $n = 4$. Messages from A are encrypted with the public key of the receiver (this encryption is not shown)

The protocol is given as follows. As before, an illustration is given for the case $n = 4$ in Figure 2.

$$1)\quad S \longrightarrow T_{a_1} \;:\; \{\, \texttt{Reveal}, \widetilde{\Phi}_A, \widetilde{\Psi}_{K_{[A]}}, S \,\}_{K_{T_{a_1}}}$$

$$2)\quad T_{a_1} \longrightarrow S \;:\; \{\, [\widetilde{\Phi}_{n-1}, \mathtt{K}_{a_1}, \widetilde{\Psi}_{K_{[A]}}]_{K^-_{T_{a_1}}} \,\}_{K_S}$$

For $i = 1$ to $n - 2$:

$$*\begin{cases} 3a)\quad S \longrightarrow T_{a_{i+1}} \;:\; \{\, \texttt{Reveal},\ ((\widetilde{\Phi}_{n-i}, \mathtt{K}_{a_i}), \ldots, (\widetilde{\Phi}_{n-1}, \mathtt{K}_{a_1}), \widetilde{\Phi}_A), \\ \qquad\qquad\qquad\qquad\qquad\qquad \widetilde{\Psi}_{K_{[A]}}, S \,\}_{K_{T_{a_{i+1}}}} \\ 4a)\quad T_{a_{i+1}} \longrightarrow S \;:\; \{\, [\widetilde{\Phi}_{n-i-1}, \mathtt{K}_{a_{i+1}}, \widetilde{\Psi}_{K_{[A]}}]_{K^-_{T_{a_{i+1}}}} \,\}_{K_S} \end{cases}$$

$$5)\quad S \longrightarrow T_{a_n} \;:\; \{\, \texttt{Reveal},\ ((\widetilde{\Phi}_1, \mathtt{K}_{a_{n-1}}), \ldots, (\widetilde{\Phi}_{n-1}, \mathtt{K}_{a_1}), \widetilde{\Phi}_A), \widetilde{\Psi}_{K_{[A]}}, S \,\}_{K_{T_{a_n}}}$$

$$6)\quad T_{a_n} \longrightarrow S \;:\; \{\, [[\, \texttt{ITKSig},\ \Phi_{n-1}, A \,]_{K_A^-}, \widetilde{\Psi}_{K_{[A]}}]_{K^-_{T_{a_n}}} \,\}_{K_S}$$

In message 3a, the tuple of $\widetilde{\Phi}_i$s serves to prevent complaint resolution messages in one session being used in another. Before sending a response each T_{a_i} checks that the sequence he receives is correct, i.e. $\{\widetilde{\Phi}_{n-i}, \mathtt{K}_{a_i}\}_{K_{T_{a_{i-1}}}}$ equals to the second element in the signed tuple $\widetilde{\Phi}_{n-i+1}$, and $\{\widetilde{\Phi}_{n-i+1}, \mathtt{K}_{a_{i-1}}\}_{K_{T_{a_{i-2}}}}$ equals to the second element in the signed tuple $\widetilde{\Phi}_{n-i+2}$, etc. , until $\widetilde{\Phi}_A$ is reached. This check ensures that T_{a_i} will not decrypt a token that is not related to $\widetilde{\Phi}_A$.

At the nth iteration S reveals the identity of the user when it receives $[\ \texttt{ITKSig},\ \Phi_{n-1}, A\ ]_{K_A^-}$ from T_{a_n}. Importantly, in the sequence of unfoldings of $\widetilde{\Phi}_{a_i}$s, S also keeps track of Φ_{a_i}s inside them, using the session keys $\mathtt{K}_{a_i}$, in order to make sure that Φ_{n-1} is formed from the session key she was given in the service request step. That is to avoid rogue token providers disrupting or misleading the identity recovery process.

3 Analysis of anonymity property

We prove that the protocol satisfies anonymity: the identity token produced by the user cannot be linked to its identity, even if all but one of the token providers are honest. We start with defining the model of the protocol, then present some general results about the static equivalence on frames involving nonces and encryptions. We conclude the section with the proof of anonymity.

3.1 Model of the protocol

The protocol is modelled in the applied π-calculus. We do not put restrictions on the number of sessions, or agents, and assume an active adversary (aka Dolev-Yao) that can inject as well as intercept messages from the network. Public

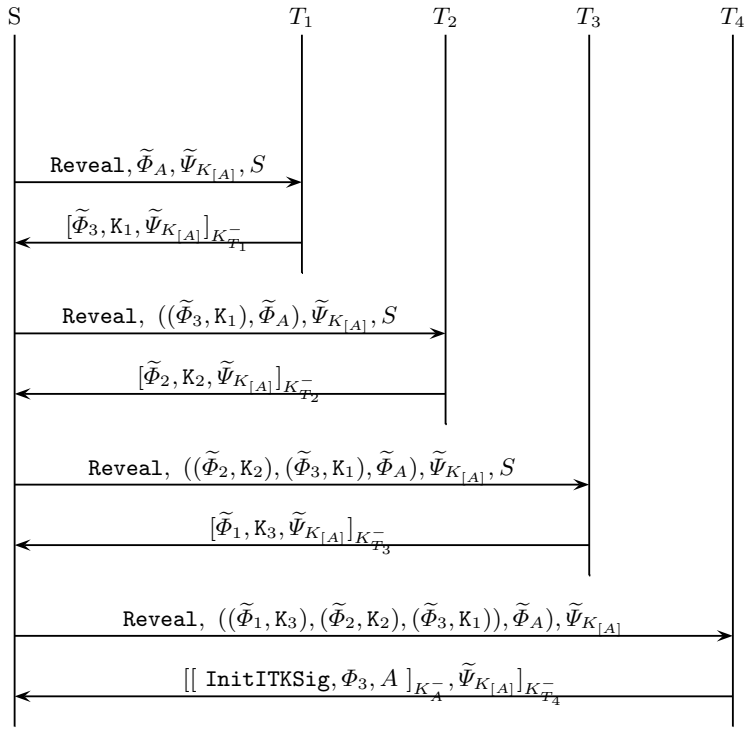

Fig. 2. Complaint resolution protocol for the case $n = 4$. All messages are encrypted with the public key of the recepient (to avoid clutter this encryption is not shown).

channels represent the network and they are the means of interacting with the environment, whereas private channels are used for private communications among processes. Channels by themselves do not reveal sender or recipient of messages, and thus are anonymous. We present the model in the language of the ProVerif tool extended with a `for`-loop construct, which is defined as a macro expansion: `for condition(j)` followed by `body(j)` stands for `body(1), body(2), ..., body(n)` , where $\{1, \ldots, n\}$ is the set of integers that satisfy the `condition(j)`. The scope of the loop is denoted by indetation (as in Python programming language).

3.2 Signature and equations

The signature of our model includes function symbols for public key cryptographic operations and universally verifiable signing, as well as other auxiliary constants and functions used in the protocol. The purpose of the functions should be clear from the comments in brackets. The equational theory is generated by the equations shown in Figure 3.

```
fun pk/1.          (* gets public key from a private key *)
fun enc/2.         (* public key encryption *)
fun senc/2.      (* symmetric key encryption *)
fun dec/2.         (* and decryption        *)
fun sdec/2.
fun sign/2.        (* universally verifiable signature *)
fun getSigKey/1.   (* retrieves public key of a signer  *)
fun getSigMess/1. (* retrieves a message from a signature *)

equation dec(enc(m,pk(sk)),sk) = m.
equation sdec(senc(m,k),k) = m.
equation getSigKey(sign(m,sk)) = pk(sk).
equation getSigMess(sign(m,sk)) = m.
```

Fig. 3. Signature and equational theory.

3.3 The protocol process

The protocol is encoded in the processes as shown in Figure 4 and 5, where `processT` and `processU` denote token provider and user, respectively (defined below).

The fresh name `skTh` represents the private decryption key of the honest token provider. In our model signing keys are different from the private decryption keys and we allow intruder to access honest token providers signing key. In the last four lines we define the protocol as the parallel composition of an arbitrary number of users and honest token provider processes. The dishonest token

providers are represented by the attacker that has an encryption key `pk(skTd)` and a signing key `signTd`.

The expression of the form `new n; P` corresponds to the restriction $\nu n.P$ of the applied-π calculus. A construct of the form `(=N,y)=M` pattern matches the left element of a tuple `M` against `N`, but assigns the right element of `M` to `y`.

3.4 Auxiliary results

Assumption 1 *Let E be an equational theory. We assume that the relation equality modulo E on terms is closed under substitutions of arbitrary terms for names and variables, and application of contexts.*

Definition 1 *We write $\{^M/_N\}$ for the substitution that replaces all occurrences of the term N by the term M. Note that $T_1 =_E T_2$ does not imply that $T_1\{^M/_N\} =_E T_2\{^M/_N\}$.*

In this section we present several results about the static equivalence of frames. Most of them are valid for the equational theory with standard public and symmetric key encryption and decryption operations, and message signing and tupling (Σ_{pk}) defined in Fig. 3. We will use the lemmas in our analysis of the anonymity property in the next section. The proofs are in appendix B.

The first simple lemma shows that exporting nonces does not affect static equivalence on frames.

Lemma 1 *Let φ, φ' be frames, $\tilde{n}, \tilde{n}'$ be sets of names and k a name s.t. $k \notin fn(\varphi, \varphi') \cap (n \cup n')$. If $\nu\tilde{n}.\varphi \approx_s \nu\tilde{n}'.\varphi'$ then $\nu\tilde{n}, k.(\{^k/_x\} \mid \varphi) \approx_s \nu\tilde{n}', k.(\{^k/_x\} \mid \varphi')$, where $x \notin dom(\varphi)$.*

The following lemma establishes sufficient conditions under which parts of frames can be simplified (substituted by fresh names). All further lemmas make use of this result.

Lemma 2 *Given an equational theory Σ, a closed term L in normal form, names $\tilde{n}, s$ and a frame φ in normal form such that $s \notin fn(\varphi)$, suppose:*

- *L does not occur in φ, and $\nu\tilde{n}.\varphi \not\vdash L$.*
- *for any $\tilde{m}, \sigma, M, N$ such that $\nu\tilde{n}.(\{^L/_x\}|\varphi) \equiv \nu\tilde{m}.\sigma$, $(fn(M) \cup fn(N)) \cap \tilde{m} = \emptyset$ and $M\sigma =_E N\sigma$ we have $M\sigma\{^z/_L\} =_E N\sigma\{^z/_L\}$.*

Then: $\nu\tilde{n}.(\{^L/_x\}|\varphi) \approx_s \nu\tilde{n}, s.(\{^s/_x\}|\varphi)$.

Lemma 3 *Let M, N and J be terms in normal form, s.t. M, N do not contain $\mathtt{dec}(x, J)$ and $M\{^{\{L\}_J}/_x\} =_E N\{^{\{L\}_J}/_x\}$, where L is in normal form. Then:*

$$(M\{^{\{L\}_J}/_x\})\{^z/_{\{L\}_J}\} =_E (N\{^{\{L\}_J}/_x\})\{^z/_{\{L\}_J}\}$$

```
free c.           (* public channel *)
free ITKReq, ITKSig, InitITKReq, InitITKSig.
free skTd,signTd. (* dishonest TP's decryption and signing keys *)
free signTh   (* honest TP's signing key *)

process
  new skTh; (* honest TP's decryption key *)
  out(c,pk(skTh)); new signA; new signB;
  let  (pkTh, pkTd) = (pk(skTh),pk(skTd)) in !processT |
  (out(c,(pk(signA),A)); let (signU,n,pU) = (signA,nA,pA) in processU) |
  (out(c,(pk(signB),B)); let (signU,n,pU) = (signB,nB,pB) in processU)


let processU=
  (* pU is Th's position in U's request chain *)
  for 0<j<n+1 & j<>pU:
     new sesK_j;
     let (upkT_j,signT_j) = (pkTd,signTd) in
  new sesK_pU;
  let (upkT_pU,signT_pU) = (pkTh,signTh) in
  new servK; out(c,pk(servK));

  (* Step 1 *)
  let phi_1=enc((InitITKReq,pk(servK),sesK_1),upkT_1) in
  (* Step 2 *)
  for 1<j<n:
    let phi_j=enc((ITKReq,phi_(j-1),sesK_j),upkT_j) in
  (* Step 3,4 *)
  let commit = sign((InitITKSig,phi_(n-1),pk(signU)),signU) in
  out(c,enc(commit,upkT_n));
  in(c,m3);
  let tphi_1=m3 in
  if getSigKey(tphi_1)=pk(signT_n) then
  let (x1, oldTphi) = getSigMess(tphi_1) in
  if oldTphi = enc((sign((ITKSig,phi_(n-1),pk(signU)),signU)),upkT_n) then
  (* Step 3a,4a *)
  for 1<j<n:
     out(c,enc((ITKSig,tphi_j,sesK_(n-j+1)),upkT_(n-j+1)));
     in(c,m4);
     let tphi_(j+1)=dec(m4,sesK_(n-j+1)) in
     if getSigKey(tphi_(j+1))=pk(signT_(n-j+1)) then
  (* Step 5,6 *)
  out(c,enc((ITKSig,tphi_n,sesK_1),upkT_1));
  in(c,m);
  let token=dec(m,sesK_1) in
  if getSigKey(token)=pk(signT_1) then
  let (x2,key) = getSigMess(token) in
  if key = servK then out(c,token)
```

Fig. 4. The Main and the user processes. We have extended ProVerif syntax with a `for`-loop (used in steps 2 and 3a,4a).

```
let processT=
  in(c,m);
  let req=dec(m,skTh) in

  let (=ITKSig,d4,k)=req in
  (
   let (x1,oldPhi) = getSigMess(d4) in
   (
   let (=InitITKReq,key,=k) = dec(oldPhi,skTh) in
   out(c,senc(sign((enc((d4,k),pkTh),key),signTh),k))
   else
   let (=ITKReq,oldPhi1,=k) = dec(oldPhi,skTh) in
   out(c,senc(sign((enc((d4,k),pkTh),oldPhi1),signTh),k))
   )
  )

  else let (=InitITKSig,d3,upk)=getSigMess(req) in
  (
   if upk=getSigKey(req) then
   out(c,sign((enc(req,pkTh),d3),signTh))
  ).
```

Fig. 5. The token provider process

All subsequent lemmas are restricted to the equiational theory with standard public and session key encryption, decryption and digital signing operations (Σ_{pk}) defined in Figure 3.

Lemma 4 *Given a frame $\nu\tilde{n}.\sigma$ in normal form that does not contain $\mathtt{dec}(x,k)$ and $\nu\tilde{n}.\sigma \not\vdash k$, then for any M, s.t. $\tilde{n} \cap fn(M) = \emptyset$, $\mathtt{dec}(x,k)$ does not occur in $M\sigma\downarrow$.*

Lemma 5 *Given a frame $\nu\tilde{n}.\sigma$ in normal form, name $k \in \tilde{n}$, s.t. k occurs in σ only as an encryption key argument, for any M, s.t. $\tilde{n} \cap fn(M) = \emptyset$, $\mathtt{dec}(x,k)$ does not occur in $M\sigma\downarrow$.*

Lemma 6 *Given a closed term L in normal form names $\tilde{n}, s$ and a frame $\nu\tilde{n}.\sigma$ in normal form, suppose $\nu\tilde{n}.\sigma \not\vdash s$ and $\{s,L\}_{pk(k)}$ does not occur in σ. Then $\nu\tilde{n}.\sigma \not\vdash \{s,L\}_{pk(k)}$.*

Lemma 7 *Given a closed term L in normal form, names $\tilde{n}, s$ and a frame $\nu\tilde{n}.\sigma$ in normal form, suppose:*

1. $\nu\tilde{n}.\sigma \not\vdash k$, $\nu\tilde{n}.\sigma \not\vdash \{s, L\}_{pk(k)}$ *and* $m \notin fn(\sigma)$
2. $\{s, L\}_{pk(k)}$ *does not occur in* σ.
3. $\mathtt{dec}(x, k)$ *does not occur in* σ.

Then $\nu\tilde{n}, s.(\{^{\{s,L\}_{pk(k)}}/_{x}\}|\sigma) \approx_s \nu\tilde{n}, m.(\{^{m}/_{x}\}|\sigma)$.

Lemma 8 *Given a normalized frame $\nu\tilde{n}.\sigma$ and $s, k \in \tilde{n}$, where $\nu\tilde{n}.\sigma \not\vdash k$, suppose for all occurences of s in σ:*

1. *either, there exists a term L such that $\{L\}_{pk(k)}$ occurs in σ and s is a subterm of L.*
2. *or s occurs in σ as an encryption key argument.*

Then $\nu\tilde{n}.\sigma \not\vdash s$.

3.5 Proof of the anonymity property

Notation and set-up:

- Th is the honest token provider and Td is one of the dishonest ones. Our aim is to show the identity token produced by the user cannot be linked to its identity, even if all but one of the token providers are honest.
- The property is shown to hold even if Th's signing key is public. We model it as a free name that intruder can use.
- We don't model dishonest token providers Td, since they form part of the attacker. Their decryption and signing keys are free names.
- Honest users A, B are instantiations of the process `processU` in Figure 4.
- $\tilde{n}_A, \tilde{n}_B$ are sets of names that include A's, B's restricted values, i.e. signing keys, service keys, and session keys generated by A, B during the run of the protocol. $\tilde{n}_A = \{K_A^-, K_{[A]}\} \cup \{\mathtt{KA}_1, \ldots, \mathtt{KA}_{n_A-1}\}$ and $\tilde{n}_B = \{K_B^-, K_{[B]}\} \cup \{\mathtt{KB}_1, \ldots, \mathtt{KB}_{n_B-1}\}$.
- $\widetilde{\Phi}(A)^j$, $\widetilde{\Phi}(B)^j$ where $j \in \{l, r\}$ denote identity tokens output by A and B, respectively.
- A's *request chain* is a sequence of token providers that A uses when building a token for anonymous service usage. It is denoted by req_A with length n_A; Th's position in the chain is p_A. Similarly, req_B, n_B, p_B refer to B's request chain.
- K_{Th} stands for a public encryption key corresponding to the decryption key sK_{Th}, i.e. $pk(sK_{Th}) = K_{Th}$.

We introduce the following notion of an oracle that is used to model anonymity.

Definition 2 *The oracle $\mathcal{R}_{Th}$ is a process that given an input m outputs a digital signature of Th on m, denoted by $[m]_{K_{Th}^-}$.*

Let A^l be the body of user A's process, such that $A = \nu \tilde{n}_A.A^l$. We define A^r to be similar to A^l with the difference being that in the former user A constructs an empty service token with the help of the oracle $\mathcal{R}_{Th}$. More precisely, A^r is the same as A^l up to the point where both receive a reply from Th. Then in A^r the user A discards the reply from Th, creates a fresh nonce ϵ and with the help of the oracle $\mathcal{R}_{Th}$ constructs the term $\widetilde{\Phi}(\varepsilon)_{p_A} = [\{\varepsilon\}_{pk(skTh)}, K_{[A]}]_{K^-_{Th}}$ if Th is the first in its request chain, or $\widetilde{\Phi}(\varepsilon)_{p_A} = [\{\varepsilon\}_{pk(skTh)}, \Phi_{n-1}]_{K^-_{Th}}$ otherwise. Then A^r continues the protocol as in A^l. All the interaction of A with the oracle is not visible to the attacker.

The resultant service token $\widetilde{\Phi}(\varepsilon)$ constructed by the user in A^r does not have A's identity embedded inside it, as opposed to the service token $\widetilde{\Phi}(A)$ constructed in A^l. So if the attacker cannot tell apart the lhs and the rhs processes depicted in the equivalence below then he cannot link the service token with the user's identity embedded inside it.

Theorem 1 *Suppose A is an honest user of the protocol, and Th is an honest token provider in A's request chain. Then the protocol guarantees user anonymity; that is,*

$$\begin{aligned} &\nu\, sK_{Th}, \tilde{n}_A.\, (A^l; \mathsf{out}(ch, \widetilde{\Phi}(A)) \mid !Th) \\ \approx\ &\nu\, sK_{Th}, \tilde{n}_A.\, (A^r; \mathsf{out}(ch, \widetilde{\Phi}(\varepsilon)) \mid !Th) \end{aligned}$$

where sK_{Th} is Th's private decryption key and ch is a public channel.

Proof. We prove labelled bisimilarity between our processes, since observational equivalence $\approx$ coincides with labelled bisimilarity $\approx_\ell$, and the latter relation is easier to reason about by hand. The definition of $\approx_\ell$ requires that every labelled and internal transitions of a process on one side of the equivalence are matched with those of a process on the other side. Furthermore, all the intermediate processes need to be statically equivalent.

In our case the matching of labelled transitions is straightforward, since we have essentially the same processes on both sides of the equivalence (only the data they manipulate are different): the Out-Atom transition only permits outputting terms by reference so we shall have the same such labels on both sides of the equivalence; and in case of In rule, the same term M will be input on both sides. We match labelled transitions as follows: for A^l's transitions on the lhs with pick those of A^r on the rhs, (and vice versa) and we match the rest with the transitions of the identical process on the other side of the equivalence.

Hence, the crux of the theorem is in proving the static equivalence of the lhs and the rhs at each step. In fact, it is sufficient to show that the largest possible frames are statically equivalent – then all subframes generated in the intermediate steps are also statically equivalent. So for our theorem we need to show the following holds:

$$\begin{aligned} &\nu\, sK_{Th}, \tilde{n}_A.(\phi^l_A \mid \{^{\widetilde{\Phi}(A)^l}/_{z_a}\} \mid (\|_{i\in N}\, \phi^l_{Th_i}) \mid \{^{K_{[A]}}/_{a_{v_n}}\} \mid \{^{K_{Th}}/_{z_3}\}) \\ \approx_s\ &\nu\, sK_{Th}, \tilde{n}_A.(\phi^r_A \mid \{^{\widetilde{\Phi}(\varepsilon)^r}/_{z_a}\} \mid (\|_{i\in N}\, \phi^r_{Th_i}) \mid \{^{K_{[A]}}/_{a_{v_n}}\} \mid \{^{K_{Th}}/_{z_3}\}) \end{aligned} \tag{1}$$

where ϕ^l_A, ϕ^r_A are A^l's and A^r's frames respectively, and $\widetilde{\Phi}(A)$ and $\widetilde{\Phi}(\varepsilon)$ represent the service tokens output by A^l and by A^r respectively. We also have $sK_{[A]}, K^-_A \in \tilde{n}_A$, and honest agent's session keys $\mathtt{KA}^j_i \in \tilde{n}_A$ for $j \in \{l, r\}$. N is a set of integers representing the number of times $!Th$ has been instantiated during the process evolution.

By inspection of the token provider process (Fig. 5), one sees that the frames it generates in response to `ITKSig` requests are of the form $\phi^j_{Th_i} = \{^{\{[\{[M_i,L_i]_{K^-_{T_i}},\mathtt{K_i}\}_{K_{Th}},L'_i]_{K^-_{Th}}\}_{\mathtt{K_i}}}/_{t_i}\}$, where $L'_i = \mathtt{snd}(\mathtt{dec}(L_i, sK_{Th}))$ and $K_i = \mathtt{thd}(\mathtt{dec}(L_i, sK_{Th}))$. From these equations, it follows that $L_i = [\{\mathtt{ITKReq}, L'_i, K_i\}_{K_{Th}}]_{K^-_{Th}}$. The frames generated by Th in response to `InitITKSig` requests are $[\{x\}_{K_{Th}}, y]_{K^-_{Th}}$, where x and y are derived from the input; since the signing key K^-_{Th} is known to the attacker in this analysis, these frames can be formed by the attacker and we need not consider them.

Let ψ_{l1} be the left-hand and ψ_{r1} be the right-hand processes of the static equivalence (1), and suppose $C_1(j)[\_]$ is a context such that $C_1(j)[\|_{i\in N}\, \phi^j_{Th_i}] = \psi_{j1}$ for $j \in \{l, r\}$. To prove (1), it is sufficient to prove

$$C_1(l)[(\mathop{\Big\|}_{i\in N_1} \phi^l_{Th_i}) \mid (\mathop{\Big\|}_{i\in N_2} \textit{dec-}\phi_{Th_i})] \approx_s C_1(r)[(\mathop{\Big\|}_{i\in N_1} \phi^r_{Th_i}) \mid (\mathop{\Big\|}_{i\in N_2} \textit{dec-}\phi_{Th_i})] \quad (2)$$

where $\textit{dec-}\phi_{Th_i} = \{^{\mathtt{dec}(L,K_{Th})}/_{t_{i_1}}\}$ and $N_1 \cup N_2 = N$. To see this, note that (1) is obtained by applying the context $\nu t_{i_1}.\_ \mid \{^{\{[\{[M,L]_{K^-_T},\mathtt{thd}(t_{i_1})\}_{x_{t_3}},\mathtt{snd}(t_{i_1})]_{K^-_{Th}}\}_{\mathtt{thd}(t_{i_1})}}/_{t_i}\}$ to each side of (2). A further step of this kind is also possible.

Let ψ_{l2} be the left-hand and ψ_{r2} be the right-hand processes of the static equivalence (2), and for $j \in \{l, r\}$ suppose $C_2(j)$ is a context such that $C_2(j)[\{^{\widetilde{\Phi}(x)^j}/_{z_a}\}] = \psi_{j2}$, where $x = A$ if $j = l$, else $x = \varepsilon$. The terms $\widetilde{\Phi}(x)^j$ have application of `sign` (by some token provider T) at their outermost level. To prove (2), it is sufficient to prove

$$\begin{aligned} &C_2(l)[\{^{\{\widetilde{\Phi}(A)^l_{n-1},\mathtt{KA}^l_{n-1}\}_{K_T}}/_{z_{a_1}}\} \mid \{^{K_{[A]}}/_{z_{a_n}}\}] \\ \approx_s\ &C_2(r)[\{^{\{y,\mathtt{KA}^r_{n-1}\}_{K_T}}/_{z_{a_1}}\} \mid \{^{K_{[A]}}/_{z_{a_n}}\})] \end{aligned} \quad (3)$$

where according to our definition of the process A^r if $K_T = K_{Th}$ then $y = \varepsilon$ else $y = \widetilde{\Phi}(\varepsilon)^r_{n-1}$. We have (3)⇒(2), because (2) is obtained by applying the context $\nu z_{a_1}, z_{a_n}, z_{b_1}, z_{b_n}.(\_ \mid \{^{[z_{a_1},z_{a_n}]_{K^-}}/_{z_a}\} \mid \{^{[z_{b_1},z_{b_n}]_{K^-}}/_{z_b}\}$, where K^- is a signing key, to each side of (3).

Intuitively, we *delayered* the term $\widetilde{\Phi}(x)^j$ by application of the closure property of the static equivalence. We recursively repeat such delayering of all non-atomic terms of the frames on both sides of the latter equivalence, except for the terms exported by the honest token provider's frames (we already dealt with Th's frames above). Delayering is performed until we reach either (i) atomic terms, or (ii) non-atomic terms which are the result of applying encryption or signing functions with a restricted name as the key argument (that intuitively represent a message encrypted with the honest token provider's public key, or A's, B's session

key, or alternatively a message signed by A or B). For example, removing one layer from $\widetilde{\Phi}(A)^l_{n-k}$ for $n-2>k>0$ results in terms $\widetilde{\Phi}(A)^l_{n-k-1}, \Phi(A)^l_k, \mathtt{KA}^l_{k+1}$ and delayering $\Phi(A)^l_k$ in turn results in terms $\Phi(A)^l_{k-1}, \mathtt{KA}^l_k$. Here is the resulting equivalence, which assumes that the honest token provider Th is at the position p_A of A's request chain req_A of the length n_A:

$$
\begin{array}{l}
C_3(l)[(||_{0<i<p_A}\{^{\mathtt{KA}^l_i}/_{a_{u_i}}\}) \mid \{^{\Phi(A)^l_{p_A}}/_{a_{u_p}}\} \mid \{^{[\Phi(A)^l_{n-1}]_{K_A^-}}/_{a_{u_n}}\} \mid \\
\{^{\{\mathtt{ITKSig},\mathtt{KA}^l_{p_A},\widetilde{\Phi}(A)^l_{n-p_A-1}\}_{K_{Th}}}/_{a_{t_5}}\}] \\
\approx_s \\
C_3(r)[(||_{0<i<p_A}\{^{\mathtt{KA}^r_i}/_{a_{u_i}}\}) \mid \{^{\{\mathtt{ITKReq},\mathtt{KA}^r_{p_A},\varepsilon\}_{K_{Th}}}/_{a_{u_p}}\} \mid \{^{[\Phi(\varepsilon)^r_{n-1}]_{K_A^-}}/_{a_{u_n}}\} \mid \\
\{^{\{\mathtt{ITKSig},\mathtt{KA}^r_{p_A},\varepsilon\}_{K_{Th}}}/_{a_{t_5}}\}]
\end{array}
\tag{4}
$$

where $C_3[\_]$ is the context $\nu sK_{Th}, \tilde{n}_A.\,(||_{i\in N_1}\,\phi^j_{Th_i} \mid ||_{i\in N_2}\,dec\text{-}\phi_{Th_i} \mid \_)$.

Remark. In the special case when $p_A = n_A$ (i.e. when Th is the last one in A's request chain) the resulting equivalence is slightly simpler and is dealt with in a similar way as below omitting non-applicable steps.

Next, we by Lemma 1 we eliminate all substitutions that export session keys. So, equivalence (4) holds if:

$$
\begin{array}{l}
\nu\, sK_{Th}, \tilde{n}'_l.(\{^{\{\mathtt{ITKReq},\,\mathtt{KA}^l_{p_A},\Phi(A)^l_{p_A-1}\}_{K_{Th}}}/_{a_{t_4}}\} \mid \{^{[\Phi(A)^l_{n-1}]_{K_A^-}}/_{a_{u_n}}\} \mid \\
\{^{\{\mathtt{ITKSig},\,\mathtt{KA}^l_{p_A},\widetilde{\Phi}(A)^l_{n-p_A-1}\}_{K_{Th}}}/_{a_{t_5}}\} \mid (||_{i\in N_1}\,\phi^l_{Th_i}) \mid (||_{i\in N_2}\,dec\text{-}\phi_{Th_i}) \mid \\
\{^{K_{Th}}/_{a_{t_3}}\} \mid \{^{K_{[A]}}/_{a_{v_n}}\}) \\
\approx_s \\
\nu\, sK_{Th}, \tilde{n}'_r.(\{^{\{\mathtt{ITKReq},\,\mathtt{KA}^r_{p_A},\varepsilon\}_{K_{Th}}}/_{x_{t_4}}\} \mid \{^{[\Phi(\varepsilon)^r_{n-1}]_{K_A^-}}/_{x_{u_n}}\} \mid \\
\{^{\{\mathtt{ITKSig},\,\mathtt{KA}^r_{p_A},\varepsilon\}_{K_{Th}}}/_{x_{t_6}}\} \mid (||_{i\in N_1}\,\phi^r_{Th_i}) \mid (||_{i\in N_2}\,dec\text{-}\phi_{Th_i}) \mid \\
\{^{K_{Th}}/_{x_{t_3}}\}) \mid \{^{K_{[A]}}/_{x_{v_n}}\}
\end{array}
\tag{5}
$$

We have unfolded $\Phi(A)^l_{p_A} = \{\mathtt{ITKReq},\, \mathtt{KA}^l_{p_A}, \Phi(A)^l_{p_A-1}\}_{K_{Th}}$, which appears in (4), to elucidate the difference between the lhs and the rhs frames.

Let ψ_{l_3}, ψ_{r_3} be the lhs and the rhs of the equivalence (5), respectively. We normalize those frames and consider each of the substitutions of the resulting equivalence in turn. In all of the cases, we start by replacing all occurrences of the exported term in question in other frames by a reference to the exporting variable:

- $\{^{\{\mathtt{ITKSig},\mathtt{KA}^j_{p_A},\widetilde{\Phi}(A)^l_{n-p_A-1}\}_{K_{Th}}}/_{x_{t_5}}\}$. This substitution resulted from delayering $\widetilde{\Phi}(A)^j_{n-p_A}$, which is a token that Th issues to A. ψ_{l_3} and ψ_{r_3} do not contain a subterm $\mathtt{dec}(x_{t_5}, sK_{Th})$ since the frames are normalized. From Th's protocol we note that it only performs decryption of messages of the form $\{\mathtt{ITKReq}, Z\}_{K_{Th}}$ for some term Z, which it receives as part of a larger input. not decrypt the message exported by x_{t_5}. …. By Lemma 8 $\psi_{j_3} \not\vdash \mathtt{KA}^j_{p_A}$. We also have for $j \in \{l, r\}$, $\psi_{j_3} \not\vdash sK_{Th}$ and since $\{\mathtt{ITKSig}, \mathtt{KA}^j_{p_A}, \widetilde{\Phi}(A)^j_{n-p_A-1}\}_{K_{Th}}$

does not occur in ψ_{j_3}, by Lemma 6 we have $\psi_{l_3}, \psi_{r_3} \not\vdash \{\mathtt{ITKSig}, \mathtt{KA}^j_{p_A}, \widetilde{\Phi}(A)^j_{n-p_A-1}\}_{K_{Th}}$. The latter observation implies that by Lemma 7, we can replace the substitution in question by a substitution of a fresh name on both sides of the equivalence.

– $\{^{\{\mathtt{ITKReq}, \mathtt{KA}^j_{Th}, \Phi(A)^j_{p_A-1}\}_{K_{Th}}}/_{x_{t_4}}\}$. This term is the token $\Phi(A)^j_{p_A}$ that honest agents produce in the construction phase of the protocol. Since ψ_{l_3}, ψ_{r_3} are normalized all occurrences of $\mathtt{dec}(\{\mathtt{KA}^j_{p_A}, \Phi(A)^j_{p_A-1}\}_{k_{Th}}, sK_{Th})$ are reduced to $\{\mathtt{KA}^j_{p_A}, \Phi(A)^j_{p_A-1}\}$, and hence the term $\mathtt{dec}(x_{t_4}, k)$ does not occur in ψ_{l_3} or ψ_{r_3}. We also have for $j \in \{l, r\}$, $\psi_{j_3} \not\vdash sK_{Th}$ and $\psi_{j_3} \not\vdash \mathtt{Kx}^j_{p_A}$ since Th encrypts all of its replies which are the decryptions of the substitution in question by $\mathtt{KA}^j_{p_A}$. Furthermore, $\{\mathtt{KA}^j_{p_x}, \Phi(A)^j_{p_A-1}\}_{K_{Th}}$ does not occur in ψ_{j_3}, and so by Lemma 6 we have $\psi_{j_3} \not\vdash \{\mathtt{KA}^j_{p_x}, \Phi(A)^l_{p_A-1}\}_{K_{Th}}$. The latter implies that as above by Lemma 7, we can replace the substitution in question by a substitution of a fresh name on both sides of the equivalence.

– $dec_{A,B}\text{-}\phi^j_{Th_i} = \{[\{\mathtt{KA}^j_{p_x}, \widetilde{\Phi}(A)^j_{n-p_A-1}\}_{K_{Th}}, \Phi(A)^j_{p_A-1}]_{K^-_{Th}}\}_{\mathtt{KA}^j_{p_A}}$. This is a reply from Th to an honest participant in the second stage of the protocol. For $j \in \{l, r\}$, we have $\mathtt{KA}_{p_A}$ occurs in ψ_{j_3} only as an encryption key argument after two transformations above, and the term encrypted by $\mathtt{KA}^j_{p_x}$ does not occur in ψ_{j_3}. Hence, by Lemma 7 we can replace the terms in question by fresh names on both sides of the equivalence.

After the above transformations we note that the lhs of the equivalence is α-equivalent to the rhs. Consequently, $\psi_{l_3} \approx_s \psi_{r_3}$.

4 Related work

In an earlier work Marshall and Molina-Jiminez [18] proposed a protocol that aimed to distribute escrowed user identity among several trusted parties using standard cryptographic primitives. However, their scheme requires strong assumptions on the trusted parties and suffers from serious flaws. In this section we briefly detail Marshall and Molina-Jiminez's (MJ) protocol and the problems we have identified with it.

The MJ protocol consists of two parts: sign-up protocol, where users generate anonymous service tokens, and complaint resolution protocol, which is executed to reveal the identity of the offending anonymous user. Those parts are depicted in Protocols 1 and 2, respectively.

In Protocol 1, E stands for a set of adjudicators $\{E_1, \ldots, E_k\}$. The set is initially chosen by S and then $H \subseteq E$ is chosen by A. The set of adjudicators H decide in case of a complaint from S whether it is valid and A's identity should be revealed. `an_id` is a pseudonym that is used by A when accessing services of S; $K_{[A]}$ is the public part of A's anonymous service key; $K_{\widehat{A}}$ is the public part of A's temporary anonymous communication key that each T_{a_i} $(i > 1)$ use to anonymously send messages to A.

In Protocol 2, Ψ is a complaint that S submits to adjudicators from the set H (chosen in the previous protocol), and to the set of token providers chosen by

1. $A \longrightarrow T_{a_1}$: $\{\ [\ \texttt{ITKReq}\ ]_{K_A^-}\ \}_{K_{T_{a_1}}}$

2. $T_{a_1} \longrightarrow A$: $\{\ \Phi_1\ \}_{K_A}$, where $\Phi_1 = [\ \{K_A\}_{K_{T_{a_1}}}\ ]_{K_{T_{a_1}}^-}$

$*$ $\begin{cases} \textsf{1a} \quad A \quad \longmapsto \quad T_{a_{i+1}} \quad : \quad \{\ \texttt{ITKSig},\ \Phi_i\ \}_{K_{T_{a_{i+1}}}} \\ \qquad\qquad\qquad\qquad \textsf{where} \quad \Phi_i = [\{\ \Phi_{i-1}\}_{K_{T_{a_i}}}\ ]_{K_{T_{a_i}}^-} \\ \textsf{2a} \quad T_{a_{i+1}} \quad \longrightarrow\!\!| \quad A \quad : \quad \{\ [\ \{\ \Phi_i\}_{K_{T_{a_{i+1}}}}\ ]_{K_{T_{a_{i+1}}}^-}\ \}_{K_{\hat{A}}} \end{cases}$

3. $A \longmapsto S$: $\{\ \texttt{ServReq}, K_{[A]}, \Phi_A\ \}_S$

4. $S \longrightarrow\!\!| A$: $\{\ n, E\ \}_{K_{[A]}}$

5. $A \longmapsto S$: $\{\ H\ \}_S$

6. $S \longrightarrow\!\!| A$: $\{\ \texttt{an\_id}\ \}_{K_{[A]}}$

Protocol 1: MJ sign-up protocol

1. $S \longrightarrow E_i$: $\{\ \texttt{AdjReq}, \Psi\ \}_{K_{E_i}}$

2. $E_i \longrightarrow S$: $\{\ [\ V_i\ ]_{K_{E_i}^-}\ \}_{K_S}$

3. $S \longrightarrow T_{a_i}$: $\{\ \texttt{Reveal}, \Phi_i, \Psi, H, V\ \}_{K_{T_{a_i}}}$

4. $T_{a_i} \longrightarrow S$: $\{\ \Phi_{i-1}\ \}_{K_S}$

Protocol 2: MJ complaint resolution protocol

A. $V = \{V_1, \ldots, V_n\}$ is the set of 'yes/no' votes by $E_i \in H$ that decide whether Ψ is valid or not. If V is positive in the majority, S presents the tuple of signed votes V to each T_{a_i}, in order to reveal the identity of a user associated with the token of the offending user.

4.1 Analysis

The protocol is subject to the following serious vulnerabilities:

Service Misuse. *Any of the identity token providers can misuse services of S (or let someone else to do that) and, furthermore, it can implicate any entity of its choice in such a misuse*:

Suppose T_{a_i} is a dishonest token provider. He can present any intermediate token which he receives during the sign-up protocol to S, and obtain an identifier to use the service. He can misuse the service and in doing so implicate the user who initiated the creation of the intermediate token. Moreover, since the identity token takes the form

$$[\ \{\ \ldots [\ \{K_A\}_{K_{T_{a_1}}}\ ]_{K^-_{T_{a_1}}}\ \ldots\}_{K_{T_{a_p}}}\ ]_{K^-_{T_{a_p}}}$$

and T_{a_1} has access to A's public key K_A, he can create $[\ \{K_A\}_{K_{T_{a_1}}}\ ]_{K^-_{T_{a_1}}}$ and anonymously request the signature services from $T_{a_2}, \ldots, T_{a_p}$ in order to create the A's full identity token Φ_A. (Other token providers cannot easily create Φ_A since they don't directly receive $[\texttt{ITKReq}]_{K^-_A}$).

Clearly, the MJ protocol requires putting full trust on all token providers that are involved in generating Φ_A, since the token is not unforgeably tied to A's real identity.

Identity Compromise (1). *Suppose A has chosen a sequence of token providers $T_{a_1}, T_{a_2}, \ldots, T_{a_p}$, and T_{a_1} is dishonest. Then the service provider S in a coalition with T_{a_1} can identify the identity token Φ_A that A has generated:*

Suppose T_{a_1} is dishonest and reveals to S identity tickets it issues. Then it takes at most n^{k-1} number of operations (`ITKSig` requests and encryptions) for the coalition to find out Φ_A, where n is the total number of identity token providers and k is the length of A's requests chain - a straightforward brute-force search. If we allow the coalition to eavesdrop locally on messages of other token providers T_{a_i}, then the number of operations they need to perform is polynomial in k, namely $n(k-1)$.

Identity Compromise (2). *Suppose S has successfully processed a complaint about a particular user. Then S can reveal the identity of any subsequent user of the service.*

Once S has successfully processed a complaint, he is in possession of the information Ψ, H, V corresponding to the complaint. He can use this to make

`Reveal` requests to any sequence of T_i's corresponding to some other protocol session, and thereby break its anonymity.

The MJ protocol has some other weaknesses too:

- Any third party can find out who misused the services of an anonymous service provider S.
- A dishonest service provider S can change the set H to include more lenient' adjudicators, when requesting to reveal the identity of an anonymous user in the complaint resolution protocol.

5 Conclusion

In the paper, we proposed a protocol that addresses the problem of anonymity with identity escrow, which is about balancing the need for anonymity with the need for traceability and accountability on the Internet. Our protocols allows online users to access the services anonymously, while providing the possibility that the service owner can break the anonymity if its service is misused.

The main feature of our scheme is it provides user anonymity even if all but one escrow holders are dishonest acting in a coalition. We analysed the anonymity property of the protocol in the applied pi calculus. We also established compelling reduction criteria for a generic public-key equational theory that stipulate conditions upon which ciphertexts (encrypted messages) reveal no more information to the attacker than fresh nonces. These results can be reused to simplify analysis elsewhere.

Lastly, we analysed a protocol proposed by Marshall and Molina-Jiminez [18] that aimed to achieve goals similar to ours, and we showed that their scheme suffers from serious flaws. We present an anonymity with identity escrow protocol that aims to provide anonymity to online users, while at the same time allowing online service providers to recover identity of offending users.

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A Applied pi-calculus

The applied pi calculus [4] is a language for describing concurrent processes and their interactions. It is based on the pi calculus, but is intended to be less pure and therefore more convenient to use. Properties of processes described in the applied pi calculus can be proved by employing manual techniques [4], or by automated tools such as ProVerif [6]. As well as reachability properties which are typical of model checking tools, ProVerif can in some cases prove that processes are observationally equivalent [7]. This capability is important for anonymity-type properties such as the one we study here. The applied pi calculus has been used to study a variety of security protocols, such as those for private authentication [12], for key establishment [3], as well as an electronic voting protocol [15].

To describe processes in the applied pi calculus, one starts with a set of *names* (which are used to name communication channels or other constants), a set of *variables*, and a *signature* Σ which consists of the function symbols which will be used to define terms. In the case of security protocols, typical function symbols will include *enc* for encryption, which takes plaintext and a key and returns the corresponding cipher text, and *dec* for decryption, taking ciphertext and a key and returning the plaintext. Terms are defined as names, variables, and function symbols applied to other terms. Terms and function symbols can be sorted, and function symbol application must then respect sorts. By the means of an equational theory E we describe the equations which hold on terms constructed from the signature. We denote $=_E$ the equivalence relation induced by E. A typical example of an equational theory useful for cryptographic protocols is

$$dec(enc(x, k), k) = x.$$

For example, given this theory and terms $T_1 = dec(enc(enc(n, k_1), k_2), k_2)$ and $T_2 = enc(n, k_1)$, we have that $T_1 =_E T_2$ (while obviously the syntactic equality

does not hold). We write $M == N$ if M is syntactically equivalent to N. By orienting the equations in the equational theory from left to right one can obtain a rewriting system R_E. If R_E is convergent than all terms have unique normal forms. $N{\downarrow}$ denotes normal form of N. The standard public key encryption equational theory Σ_{pk} which includes projections, pairing, digital signing, public and session key encyption, decryption functions is known to be convergent.

In the applied pi calculus, one has (plain) processes and extended processes. Plain processes are built up in a similar way to processes in the pi calculus, except that messages can contain terms (rather than just names). In the grammar described below, M and N are terms, n is a name, x a variable and u is a metavariable, *i.e.* a name or a variable.

$P, Q, R :=$	plain processes
0	null process
$P \mid Q$	parallel composition
$!P$	replication
$\nu n.P$	name restriction
if $M = N$ then P else Q	conditional
$\mathsf{in}(u, x).P$	message input
$\mathsf{out}(u, N).P$	message output

Extended processes add *active substitutions* and restriction on variables:

$A, B, C :=$	extended processes
P	plain process
$A \mid B$	parallel composition
$\nu n.A$	name restriction
$\nu x.A$	variable restriction
$\{^M/_x\}$	active substitution

$\{^M/_x\}$ is the substitution that replaces the variable x with the term M. Active substitutions generalise "let". The process $\nu x.(\{^M/_x\} \mid P)$ corresponds exactly to "let $x = M$ in P". As usual, names and variables have scopes, which are delimited by restrictions and by inputs. We write $fv(A)$, $bv(A)$, $fn(A)$ and $bn(A)$ for the sets of free and bound variables and free and bound names of A, respectively. We say that an extended process is closed if all its variables are either bound or defined by an active substitution.

Active substitutions are useful because they allow us to map an extended process A to its *frame* $\phi(A)$ by replacing every plain process in A with 0. A frame is an extended process built up from 0 and active substitutions by parallel composition and restriction. The frame $\phi(A)$ can be viewed as an approximation of A that accounts for the static knowledge A exposes to its environment, but not A's dynamic behaviour. Every frame can be written in the form $\varphi = \nu\tilde{n}.\{T_1/x_1, \ldots, T_n/x_n\}$. The domain of φ is $dom(\varphi) = \{x_1, \ldots, x_n\}$. We say that φ is closed if all the terms T_i are closed, *i.e.* contain no variables.

Definition 3 *We say two terms M, N are equal in the frame φ, and write $(M = N)\varphi$, if $\varphi \equiv \nu\tilde{n}.\sigma$, $M\sigma =_E N\sigma$, and $\tilde{n} \cap fn(M, N) = \emptyset$ for some names $\tilde{n}$ and substitution σ.*

A context $C[\cdot]$ is a process with a hole; an evaluation context is a context whose hole is not under a replication, a conditional, an input, or an output. Structural equivalence, noted $\equiv$, is the smallest equivalence relation on extended processes that is closed under α-conversion on names and variables, by application of evaluation contexts, and satisfying some further basic structural rules such as $A \mid 0 \equiv A$, associativity and commutativity of $\mid$, binding-operator-like behaviour of ν, and when $M =_E N$ the equivalences:

$$\nu x.\{^M/_x\} \equiv 0 \qquad \{^M/_x\} \equiv \{^N/_x\}$$
$$\{^M/_x\} \mid A \equiv \{^M/_x\} \mid A\{^M/_x\} \ .$$

We can now define what it means for two frames to be statically equivalent.

Definition 4 (Static equivalence) *Two closed frames φ_1 and φ_2 are* statically equivalent*, written $\varphi_1 \approx_s \varphi_2$, iff for some names $\tilde{n}_1$, $\tilde{n}_2$ and substitutions σ_1, σ_2, such that $\varphi_1 \equiv \nu\tilde{n}_1.\sigma_1$ and $\varphi_2 \equiv \nu\tilde{n}_2.\sigma_2$, we have that:*

(i) $\mathrm{dom}(\varphi_1) = \mathrm{dom}(\varphi_2)$,
(ii) for all terms M, N with variables included in $\mathrm{dom}(\varphi_i)$ *and using no names occurring in $\tilde{n}_1$ or $\tilde{n}_2$, $M\sigma_1 =_E N\sigma_1$ is equivalent to $M\sigma_2 =_E N\sigma_2$.*

We also say that two extended processes A and B are statically equivalent if and only if $\phi(A) \approx_s \phi(B)$.

Example 1 *Consider $\varphi_0 = \nu k.\{enc(s_0, k)/x_1, k/x_2\}$ and $\varphi_1 = \nu k.\{enc(s_1, k)/x_1, k/x_2\}$ where s_1, s_2 and k are names. Let E be the theory defined by the axiom $dec(enc(x, k), k) = x$. We have $(dec(x_1, x_2) =_E s_0)\varphi_0$ but not $(dec(x_1, x_2) =_E s_0)\varphi_1$. Therefore, $\varphi_0 \not\approx_s \varphi_1$ although $\nu k.\{enc(s_0, k)/x_1\} \approx_s \nu k.\{enc(s_1, k)/x_1\}$.*

The operational semantics of processes in the applied pi calculus is defined by structural rules defining two relations: *structural equivalence* (described above) and *internal reduction*, noted $\rightarrow$. Internal reduction $\rightarrow$ is the smallest relation on extended processes closed under structural equivalence and application of evaluation contexts such that $\bar{a}\langle x\rangle.P \mid a(x).Q \rightarrow P \mid Q$ and whenever $M \neq_E N$,

$$\text{if } M = M \text{ then } P \text{ else } Q \rightarrow P$$
$$\text{if } M = N \text{ then } P \text{ else } Q \rightarrow Q.$$

Definition 5 (Deducibility) *Given a frame φ that represents a history of messeages output during the execution of the processes, we define the* deduction *relation, denoted by $\varphi \vdash M$. Deducible messages are those that can be derived from φ by application of function symbols and the equational theory E.*

$$\frac{}{\nu\tilde{n}.\,\sigma \vdash x\sigma}[x \in dom(\sigma)] \qquad \frac{}{\nu\tilde{n}.\,\sigma \vdash m}[m \in \mathcal{N} \setminus \tilde{n}]$$

$$\frac{\nu\tilde{n}.\,\sigma \vdash T_1 \quad \ldots \quad \nu\tilde{n}.\,\sigma \vdash T_l}{\nu\tilde{n}.\,\sigma \vdash f(T_1, \ldots, T_l)} \qquad \frac{\nu\tilde{n}.\,\sigma \vdash T \qquad T =_E T'}{\nu\tilde{n}.\,\sigma \vdash T'}$$

Example 2 *n is deducible from the frame $\nu n, k'.(\{^{enc(n,k')}/_x\} \mid \{^{k'}/_y\})$, but not from the frame $\nu n.\{^{f(n)}/_x\}$, where f is a hash function.*

Proposition 1. *Let $\varphi = \nu\tilde{n}.\sigma$ be a frame and M be a term. $\varphi \vdash M$ if and only if there exists a term T, such that $fn(T) \cap \tilde{n} = \emptyset$ and $T\sigma =_E M$.*

We further extend the operational semantics by a labeled operational semantics enabling us to reason about processes that interact with their environment. Labeled operational semantics defines the relation $\xrightarrow{\alpha}$ where α is either an input or the output of a channel name or a variable of base type. More details can be found in [4].

$$\mathsf{in}(c, x).P \xrightarrow{\mathsf{in}(c,M)} P\{^M/_x\} \quad \textsc{In} \qquad \mathsf{out}(c, u).P \xrightarrow{\mathsf{out}(c,u)} P \quad \textsc{Out-Atom}$$

$$\frac{A \xrightarrow{\mathsf{out}(c,u)} A'}{\nu u.A \xrightarrow{\nu u.\mathsf{out}(c,u)} A'}[\textsc{Open-Atom}] \qquad \frac{A \xrightarrow{\alpha} A'}{\nu u.A \xrightarrow{\alpha} \nu u.A'}[\textsc{Scope}]$$

$$\frac{A \xrightarrow{\alpha} A'}{A \mid B \xrightarrow{\alpha} A' \mid B}[\textsc{Par}] \qquad \frac{A \equiv B \quad B \xrightarrow{\alpha} B' \quad B' \equiv A'}{A \xrightarrow{\alpha} A'}[\textsc{Struct}]$$

where u is a metavariable that ranges over names and variables, and Par rule is restricted to $bv(\alpha) \cap fv(B) = bn(\alpha) \cap fn(B) = \emptyset$.

Definition 6 (Labeled bisimilarity ($\approx_\ell$)) *Labeled bisimilarity is the largest symmetric relation $\mathcal{R}$ on closed extended processes, such that $A\ \mathcal{R}\ B$ implies:*

1. *$A \approx_s B$;*
2. *if $A \to A'$, then $B \to^* B'$ and $A'\ \mathcal{R}\ B'$ for some B';*
3. *if $A \xrightarrow{\alpha} A'$, then $B \to^* \xrightarrow{\alpha} \to^* B'$ and $A'\ \mathcal{R}\ B'$ for some B'.*

In [4], it is shown that labeled bisimilarity coincides with observational equivalence, which is used to formalize many security properties, and in particular anonymity properties. In this work we prefer to use labeled bisimilarity, rather than observational equivalence, because proofs for labeled bisimilarity are easier for us to carry out.

B Proofs of lemmas

Lemma 1 *Let φ, φ' be frames, $\tilde{n}, \tilde{n}'$ be sets of names and k a name s.t. $k \notin fn(\varphi, \varphi') \cap (n \cup n')$. If $\nu\tilde{n}.\varphi \approx_s \nu\tilde{n}'.\varphi'$ then $\nu\tilde{n}, k.(\{^k/_x\} \mid \varphi) \approx_s \nu\tilde{n}', k.(\{^k/_x\} \mid \varphi')$, where $x \notin dom(\varphi)$.*

Proof. Let $\phi_l \equiv \nu\tilde{m}_l.\sigma_l$ and $\phi_r \equiv \nu\tilde{m}_r.\sigma_r$ be the lhs and the rhs of the proposed equivalence, respectively. Suppose $\nu\tilde{n}.\varphi \equiv \nu\tilde{m}.\sigma$ with $\tilde{m} = \tilde{m}_l \setminus \{k\}$, since by our hypothesis $k \notin fn(\varphi, \varphi') \cap (n \cup n')$, and let $\nu\tilde{n}.\varphi \approx_s \nu\tilde{n}'.\varphi'$. We have $\sigma_l = \sigma \mid \{^k/_x\}$.

Take two terms M, N such that $fn(M, N) \cap \tilde{m}_l = \emptyset$. If $(M = N)\phi_l$ then $M\sigma_l =_E N\sigma_l$, and so $(M\sigma)\{^k/_x\} =_E (N\sigma)\{^k/_x\}$. Since $fv(\sigma) = \emptyset$ and $x \notin dom(\varphi)$, we have $(M\sigma)\{^k/_x\} == (M\{^k/_x\})\sigma$, $(N\sigma)\{^k/_x\} == (N\{^k/_x\})\sigma$, and hence $(M\{^k/_x\})\sigma =_E (N\{^k/_x\})\sigma$. As $k \notin \tilde{m}$ we get $((M\{^k/_x\}) = (N\{^k/_x\}))\nu\tilde{n}.\varphi$.

Now, if $(M = N)\nu\tilde{n}.\varphi$ then $(M\sigma)\{^k/_x\} = (N\sigma)\{^k/_x\}$, and hence $(M = N)\phi_l$, since equational theory is closed under substitution of arbitrary terms for variables. So, $(M = N)\phi_l$ iff $(M = N)\nu\tilde{n}.\varphi$ and similarly we have $(M = N)\phi_r$ iff $((M\{^k/_x\}) = (N\{^k/_x\}))\nu\tilde{n}'.\varphi'$, which imply $(M = N)\phi_l$ iff $(M = N)\phi_r$. Hence, $\phi_l \approx_s \phi_r$.

Lemma 2 *Given an equational theory Σ, a closed term L in normal form, names $\tilde{n}, s$ and a frame φ in normal form such that $s \notin fn(\varphi)$, suppose:*

- *L does not occur in φ, and $\nu\tilde{n}.\varphi \not\vdash L$.*
- *for any $\tilde{m}, \sigma, M, N$ such that $\nu\tilde{n}.(\{^L/_x\}|\varphi) \equiv \nu\tilde{m}.\sigma$, $(fn(M) \cup fn(N)) \cap \tilde{m} = \emptyset$ and $M\sigma =_E N\sigma$ we have $M\sigma\{^z/_L\} =_E N\sigma\{^z/_L\}$.*

Then: $\nu\tilde{n}.(\{^L/_x\}|\varphi) \approx_s \nu\tilde{n}, s.(\{^s/_x\}|\varphi)$.

Proof. Let ϕ_l, ϕ_r denote the right and left parts of the equivalence, respectively. We need to show that for any terms J, K we have $(J = K)\phi_l$ iff $(J = K)\phi_r$.

"⇒" direction. Suppose $(J = K)\phi_l$. Then, $\exists\tilde{m}, \sigma$ such that $\phi_l \equiv \nu\tilde{m}.\sigma$, $J\sigma =_E K\sigma$ and $\tilde{m} \cap (fn(J) \cup fn(K)) = \emptyset$. By hypothesis, $(J\sigma)\{^z/_L\} =_E (K\sigma)\{^z/_L\}$, and hence, $((J\sigma)\{^z/_L\})\{^s/_z\} =_E ((K\sigma)\{^z/_L\})\{^s/_z\}$, where $s \notin fn(K) \cup fn(J)$.

Wlog suppose φ is a substitution σ_φ, then $\sigma = \sigma_\varphi \cup \{^L/_x\}$. Consider terms $J\sigma_\varphi$ and $K\sigma_\varphi$. Since L does not occur in φ and $\nu\tilde{n}.\varphi \not\vdash L$, L does not occur in $J\sigma_\varphi$ and $K\sigma_\varphi$. Hence, $((J\sigma)\{^z/_L\})\{^s/_z\} = (((J\sigma_\varphi)\{^L/_x\})\{^z/_L\})\{^s/_z\} = (J\sigma_\varphi)\{^s/_x\}$, and similarly $((K\sigma)\{^z/_L\})\{^s/_z\} = (K\sigma_\varphi)\{^s/_x\}$. We have $\phi_r \equiv \nu\tilde{m}'.\sigma'$, where $\tilde{m}' \cap fn(J, K) = \emptyset$ and $\sigma' = \sigma_\varphi \cup \{^s/_x\}$. Therefore, $(J = K)\phi_r$.

"⇐" direction. Suppose $(J = K)\phi_r$. Then $\exists\tilde{m}, \sigma$ such that $\phi_r \equiv \nu\tilde{m}.\sigma$, $J\sigma =_E K\sigma$ and $\tilde{m} \cap fn(J, K) = \emptyset$. Wlog suppose φ is a substitution σ_φ, then $\sigma = \sigma_\varphi \cup \{^s/_x\}$, where $s \in \tilde{m}$, and $(J\sigma_\varphi)\{^s/_x\} =_E (K\sigma_\varphi)\{^s/_x\}$, and also $J\sigma_\varphi =_E K\sigma_\varphi$ by our assumption 1 that equational theory is closed under substitution

of arbitrary terms for names. So, $(J\sigma_\varphi)\{^L/_x\} =_E (K\sigma_\varphi)\{^L/_x\}$. We have $\phi_l \equiv \nu\tilde{m}'.\sigma'$, where $\tilde{m}' \cap fn(J,K) = \emptyset$ and $\sigma' = \sigma_\varphi\{^L/_x\}$. Therefore, $(J = K)\phi_l$.

Lemma 3 *Let M, N and J be terms in normal form, s.t. M, N do not contain $\mathtt{dec}(x, J)$ and $M\{^{\{L\}_J}/_x\} =_E N\{^{\{L\}_J}/_x\}$, where L is in normal form. Then:*

$$(M\{^{\{L\}_J}/_x\})\{^z/_{\{L\}_J}\} =_E (N\{^{\{L\}_J}/_x\})\{^z/_{\{L\}_J}\}$$

Proof. Since M, L and J are in normal form and $\mathtt{dec}(x, k)$ does not occur in M, the term $M\{^{\{L\}_J}/_x\}$ is in normal form. Similarly, $N\{^{\{L\}_J}/_x\}$ is also in normal form. We have $M\{^{\{L\}_J}/_x\} =_E N\{^{\{L\}_J}/_x\}$ and since normal forms are unique in Σ_{pk}, we have $M\{^{\{L\}_J}/_x\} == N\{^{\{L\}_J}/_x\}$.

Lemma 4 *Given a frame $\nu\tilde{n}.\sigma$ in normal form that does not contain $\mathtt{dec}(x, k)$ and $\nu\tilde{n}.\sigma \not\vdash k$, then for any M, s.t. $\tilde{n} \cap fn(M) = \emptyset$, $\mathtt{dec}(x, k)$ does not occur in $M\sigma\downarrow$.*

Proof. The proof is by induction on M.

Base case: $M == u$. If $u \notin dom(\sigma)$ then nothing to do, else $u\sigma == N$, where $\{^N/_u\}$ is a substitution in σ. N is in normal form, since σ is in normal form, and it does not contain $\mathtt{dec}(x, k)$ as $\mathtt{dec}(x, k)$ does not occur in σ.

Induction step. Suppose for some M, N, $\tilde{n} \cap fn(M, N) = \emptyset$, $\mathtt{dec}(x, k)$ does not occur in $M\sigma\downarrow$ and $N\sigma\downarrow$. Then we need to consider the following cases:

- $M' == f(M, N)$, where f is $\mathtt{enc}, \mathtt{pair}$ or $\mathtt{sign}$: we have $M'\sigma\downarrow == f(M\sigma\downarrow, N\sigma\downarrow)$. So, by the IH $\mathtt{dec}(x, k)$ does not occur in $M'\sigma\downarrow$.
- $M' == f(M)$, where f is a unary function in Σ_{pk}. $M'\sigma\downarrow$ is $f(M\sigma\downarrow)$ or M'', where M'' is a subterm of $M\sigma\downarrow$ or $pk(N)$ for some subterm N of $M\sigma\downarrow$. By our IH $\mathtt{dec}(x, k)$ does not occur in $M'\sigma\downarrow$.
- $M' == \mathtt{dec}(M, N)$. $M'\sigma\downarrow == \mathtt{dec}(M\sigma\downarrow, N\sigma\downarrow)$ or $M'\sigma\downarrow$ is a subterm of $M\sigma\downarrow$. The term $N\sigma\downarrow$ is distinct from k, since $\nu\tilde{n}.\sigma \not\vdash k$. Again, by the IH $\mathtt{dec}(x, k)$ does not occur in $M'\sigma\downarrow$.

Lemma 5 *Given a frame $\nu\tilde{n}.\sigma$ in normal form, name $k \in \tilde{n}$, s.t. k occurs in σ only as an encryption key argument, for any M, s.t. $\tilde{n} \cap fn(M) = \emptyset$, $\mathtt{dec}(x, k)$ does not occur in $M\sigma\downarrow$.*

Proof. The proof is by induction on M.

Base case: $M == u$. If $u \notin dom(\sigma)$ then nothing to do, else $u\sigma == N$, where $\{^N/_u\}$ is a substitution in σ. N is in normal form, since σ is in normal form, and it does not contain $\mathtt{dec}(x, k)$ by our assumption.

Induction step. Suppose for some M, N, $\tilde{n} \cap fn(M, N) = \emptyset$, $\mathtt{dec}(x, k)$ does not occur in $M\sigma\downarrow$ and $N\sigma\downarrow$. Then we need to consider the following cases:

– $M' == f(M, N)$, where f is $\mathtt{enc}, \mathtt{pair}$ or $\mathtt{sign}$: we have $M'\sigma\downarrow == f(M\sigma\downarrow, N\sigma\downarrow)$. So, by the IH $\mathtt{dec}(x, k)$ does not occur in $M'\sigma\downarrow$.
– $M' == f(M)$, where f is a unary function in Σ_{pk}. $M'\sigma\downarrow$ is $f(M\sigma\downarrow)$, M'', or $pk(M'')$, where M'' is a subterm of $M\sigma\downarrow$. By our IH $\mathtt{dec}(x, k)$ does not occur in $M'\sigma\downarrow$.
– $M' == \mathtt{dec}(M, N)$. $M'\sigma\downarrow == \mathtt{dec}(M\sigma\downarrow, N\sigma\downarrow)$ or $M'\sigma\downarrow$ is a subterm of $M\sigma\downarrow$. The term $N\sigma\downarrow$ is distinct from k, since $\nu\tilde{n}.\sigma \nvdash k$, which easily follows from our assumption that k occurs in σ only as an encryption key argument. So, again by the IH $\mathtt{dec}(x, k)$ does not occur in $M'\sigma\downarrow$.

Lemma 6 *Given a closed term L in normal form names $\tilde{n}, s$ and a frame $\nu\tilde{n}.\sigma$ in normal form, suppose $\nu\tilde{n}.\sigma \nvdash s$ and $\{s, L\}_{pk(k)}$ does not occur in σ. Then $\nu\tilde{n}.\sigma \nvdash \{s, L\}_{pk(k)}$.*

Proof. Let $L' = \{s, L\}_{pk(k)}$. It is known that $\nu\tilde{n}.\sigma \nvdash L'$ if for any M, s.t. $fn(M) \cap \tilde{n} = \emptyset$, $M\sigma \neq_E L'$. The latter inequality will hold if we show that $M\sigma\downarrow \neq_E L'$. We proceed by induction on M.

Base case: $M == u$. If $u \notin dom(\sigma)$ then nothing to do, else $u\sigma == N$, where $\{^N/_u\}$ is a substitution in σ. As σ is in normal form, all subterms of N are also in normal form. We note that L' is in normal form too because L is in normal form by assumption. Furthermore, every subterm K of N is distinct from L', as L' does not occur in σ. As a result, for all such K, $K \neq_E L'$ by the fact that normal forms are unique.

Induction step. Take some M, N, $\tilde{n} \cap fn(M, N) = \emptyset$, and suppose for all terms K, s.t. K is a subterm of $M\sigma\downarrow$ or it is a subterm of $N\sigma\downarrow$, we have $K \neq_E L'$. Then we need to consider the following cases:

– $M' == f(M, N)$, where f is $\mathtt{pair}$ or $\mathtt{sign}$: we have $M'\sigma\downarrow == f(M\sigma\downarrow, N\sigma\downarrow)$. So, by the IH for all subterms K of $M'\sigma\downarrow$ we get $K \neq_E L'$.
– $M' == \mathtt{dec}(M, N)$: $M'\sigma\downarrow == \mathtt{dec}(M\sigma\downarrow, N\sigma\downarrow)$ or it is M'', where M'' is a subterm of $M\sigma\downarrow$. In any case, by the IH for all subterms K of $M'\sigma\downarrow$ we get $K \neq_E L'$.
– $M' == f(M)$, where f is a unary function in Σ_{pk}. $M'\sigma\downarrow$ is $f(M\sigma\downarrow)$, M'' or $pk(M'')$, where M'' is a subterm of $M\sigma\downarrow$. By our IH for all subterms K of $M'\sigma\downarrow$ we get $K \neq_E L'$.
– $M' == \mathtt{enc}(M, N)$. $M'\sigma\downarrow == \mathtt{enc}(M\sigma\downarrow, N\sigma\downarrow)$. The term $M\sigma\downarrow \neq_E \{s, L\}$, since otherwise $\mathtt{fst}(M\sigma\downarrow) =_E s$ contradicting our assumption $\nu\tilde{n}.\sigma \nvdash s$. So, again by the IH for all subterms K of $M'\sigma\downarrow$ we get $K \neq_E L'$.

Lemma 7 *Given a closed term L in normal form, names $\tilde{n}, s$ and a frame $\nu\tilde{n}.\sigma$ in normal form, suppose:*

1. *$\nu\tilde{n}.\sigma \not\vdash k$, $\nu\tilde{n}.\sigma \not\vdash \{s, L\}_{pk(k)}$ and $m \notin fn(\sigma)$*
2. *$\{s, L\}_{pk(k)}$ does not occur in σ.*
3. *$\mathtt{dec}(x, k)$ does not occur in σ.*

Then $\nu\tilde{n}, s.(\{^{\{s,L\}_{pk(k)}}/_{x}\}|\sigma) \approx_s \nu\tilde{n}, m.(\{^{m}/_{x}\}|\sigma)$.

Proof. Take terms M, N such that $fn(M, N) \cap \tilde{n} = \emptyset$ and $M\sigma' =_E N\sigma'$, where $\sigma' = \sigma \cup \{^{\{s,L\}_{pk(k)}}/_{x}\}$. We show that any $M\sigma'\{^{z}/_{\{s,L\}_{pk(k)}}\} =_E N\sigma'\{^{z}/_{\{s,L\}_{pk(k)}}\}$.

The substitution σ is in normal form and it does not contain $\mathtt{dec}(x, k)$ by assumption 3. Since $\nu\tilde{n}.\sigma \not\vdash k$ by Lemma 4 we have $M\sigma\downarrow$, $N\sigma\downarrow$ do not contain $\mathtt{dec}(x, k)$. The term $\{s, L\}_{pk(k)}$ is in normal form, since L is in normal form by assumption. As a result, by Lemma 3 $((M\sigma\downarrow)\{^{\{s,L\}_{pk(k)}}/_{x}\})\{^{z}/_{\{s,L\}_{pk(k)}}\} =_E ((N\sigma\downarrow)\{^{\{s,L\}_{pk(k)}}/_{x}\})\{^{z}/_{\{s,L\}_{pk(k)}}\}$, and hence $((M\sigma)\{^{\{s,L\}_{pk(k)}}/_{x}\})\{^{z}/_{\{s,L\}_{pk(k)}}\} =_E ((N\sigma)\{^{\{s,L\}_{pk(k)}}/_{x}\})\{^{z}/_{\{s,L\}_{pk(k)}}\}$.

By our assumptions $\nu\tilde{n}.\varphi \not\vdash \{s, L\}_{pk(k)}$ and $\{s, L\}_{pk(k)}$ does not occur in σ, so by Lemma 2 it follows that our lemma holds.

Lemma 8 *Given a normalized frame $\nu\tilde{n}.\sigma$ and $s, k \in \tilde{n}$, where $\nu\tilde{n}.\sigma \not\vdash k$, suppose for all occurences of s in σ:*

1. *either, there exists a term L such that $\{L\}_{pk(k)}$ occurs in σ and s is a subterm of L.*
2. *or s occurs in σ as an encryption key argument.*

Then $\nu\tilde{n}.\sigma \not\vdash s$.

Proof. Take terms M, N such that $fn(M, N) \cap \tilde{m} = \emptyset$ and $M\sigma' =_E N\sigma'$, where $\sigma' = \sigma\{^{\{s,L\}_{pk(k)}}/_{x}\}$. We show that $M\sigma'\{^{z}/_{\{L\}_k}\} =_E N\sigma'\{^{z}/_{\{L\}_k}\}$.

The substitution σ is in normal form, where k occurs only as an encryption key argument so by Lemma 5 $M\sigma\downarrow$, $N\sigma\downarrow$ do not contain $\mathtt{dec}(\mathtt{x}, \mathtt{k})$. Since L is in normal form using Lemma 3 we conclude $M\sigma'\{^{z}/_{\{L\}_k}\} =_E N\sigma'\{^{z}/_{\{L\}_k}\}$.

From our assumptions 1 and 2 it follows that $\nu\tilde{n}.\sigma \not\vdash \{L\}_k$ and $\{L\}_k$ does not occur in σ, so by Lemma 2 our lemma holds.